Improving the Speed and Performance of Adaptive Equalizers via Transform Based Adaptive Filtering

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Agenda

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Convergence of Adaptive Equalizers

- Convergence rate of the weights depends on the conditioning of the autocorrelation matrix of the input.
- The MSE of an adaptive filter trained with LMS decreases as a sum of exponentials whose time constants are inversely proportional to the eigenvalues of the autocorrelation matrix.
- Best convergence occurs when all eigenvalues of the autocorrelation matrix are equal. Want an autocorrelation matrix that is proportional to the identity matrix.

Transform LMS Algorithm Theory

- Apply a Transform LMS (TLMS) algorithm to improve convergence times. Transform is fixed and data-independent.
- Performance is based on orthogonalizing capability of the transform used to preprocess the input
- Continuously transforming the input samples decorrelates them => inputs have equal power
- Transformation and power normalization causes eigenvalues of the inputs to cluster around one value => faster convergence of adaptive weights

TLMS Intuitive Approach

- Apply transform on the input

$$u_k(i) = \sum_{l=0}^{n-1} T_n(i,l)\, x_{k-l}$$

- Perform power normalization

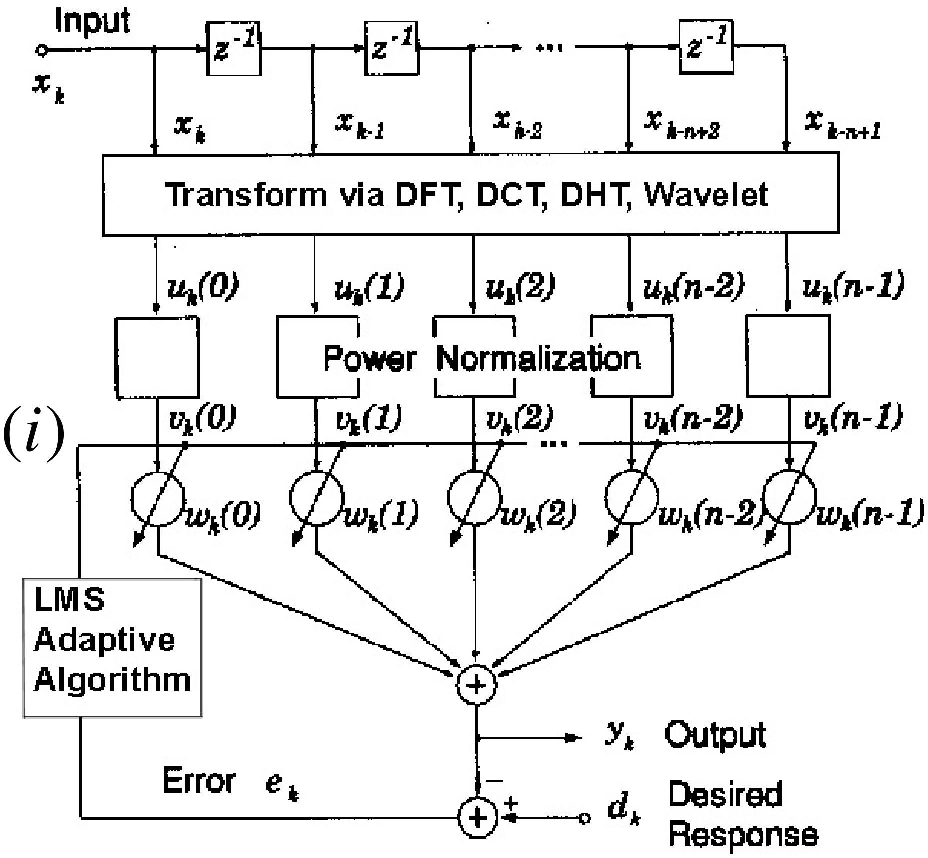
$$v_k(i) = \frac{u_k(i)}{\sqrt{P_k(i) + \varepsilon}}$$

$$P_k(i) = \beta P_{k-1}(i) + (1-\beta)\, u_k^2(i)$$

- Calculate error and weights (LMS coefficients)

$$e_k = d_k - \sum_{i=0}^{n-1} v_k(i)\, w_k(i)$$

$$w_{k+1}(i) = w_k(i) + \mu e_k v_k^*(i)$$



TLMS Algorithm wrt Autocorrelation

Apply the transform to the input.

$$S_k = TX_k$$

Transform the autocorrelation matrix of the input, R to obtain Rs.

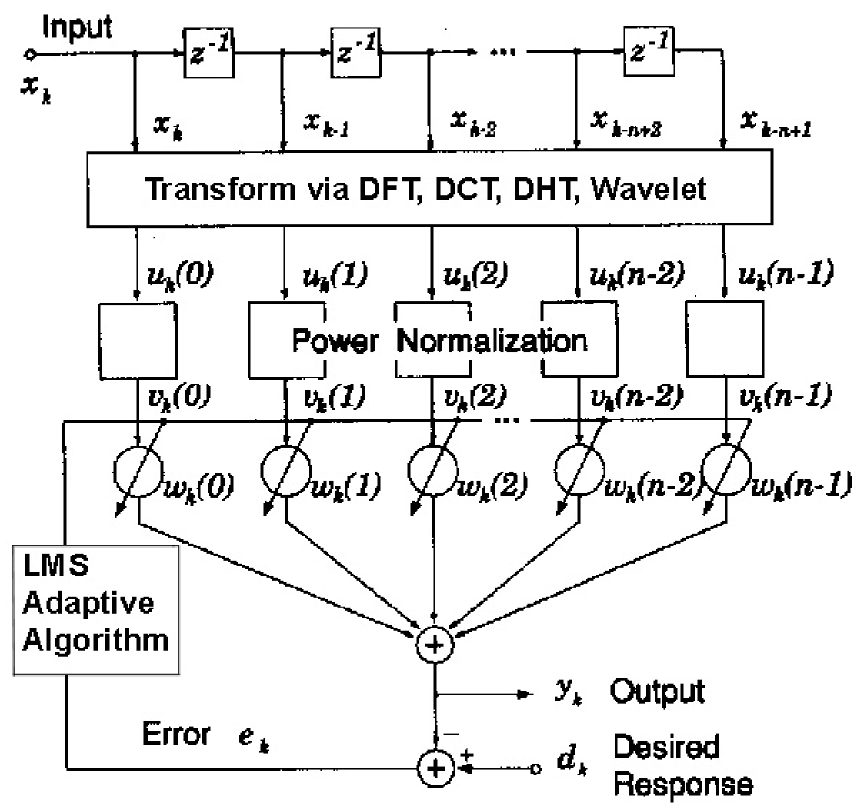
$$R_s = TRT^T$$

Sk and Rs are used in the TLMS algorithm to update the coefficients

$$W_{s_{k+1}} = W_{s_k} + 2\mu_s e_k S_k R_s^{-1}$$

The inverse autocorrelation matrix is approximated by a diagonal matrix MxM with elements that estimate the power of each of the transform vector's Sk components

$$R_s^n(k,k)^{-1} = \beta R_s^{n-1}(k,k)^{-1} + (1-\beta) S_k^n(k)^2$$



Some Possible Transforms

Discrete Cosine Transform (DCT)	$T_n(i,l) = C_n(i,l) = \sqrt{\frac{2}{n}} K_i \cos\left(\frac{i\left(l+\frac{1}{2}\right)\pi}{n}\right)$
Discrete Fourier Transform (DFT)	$T_n(i,l) = F_n(i,l) = \sqrt{\frac{1}{n}} e^{j(2\pi il/n)}$
Discrete Hartley Transform (DHT)	$T_n(i,l) = H_n(i,l) = \sqrt{\frac{1}{n}}\left(\cos\left(\frac{2\pi il}{n}\right) + \sin\left(\frac{2\pi il}{n}\right)\right)$
Discrete Wavelet Transform (DWT)	Symmlet, Haar, Daubechies

Simulation: Raised Cosine Channel

Pre-filter Post-filter and channel combined response

$$C_{rc}[n] = \begin{cases} (1/2)\left[1+\cos\left(\frac{2\pi}{3.97}(n-2)\right)\right], n = 1,2,3 \\ 0 \qquad ,otherwise \end{cases}$$

Eigenvalue ratio = max eigenvalue / minimum eigenvalue

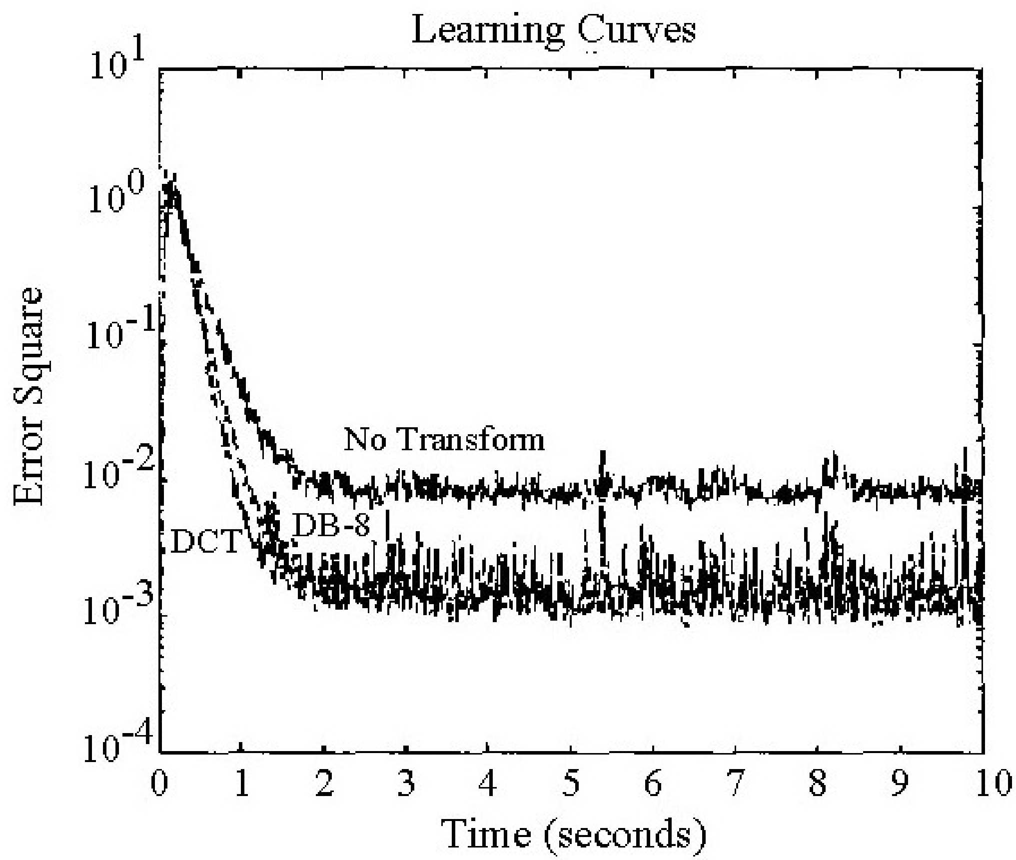
Ideally, want Eigenvalue Ratio = 1

Transform	Eigenvalue Ratio
Haar*	245.5
DCT	2.9
DFT	192.3
DHT	188.3
Daubachies 4*	159.1
Daubachies 8*	92.3
Symmlet*	85.6
No Transform	2320.5

Simulation: A Realistic Channel

$Cch(z) = 0.05 - 0.063z^{-1} + 0.088z^{-2} - 0.126z^{-3} - 0.25z^{-4} + 0.9047z^{-5} + 0.25z^{-6} + 0.126z^{-7} + 0.038z^{-8} + 0.088z^{-9}$

Transform	Eigenvalue Ratio
Haar*	1.3
DCT	1.2
DFT	1.3
DHT	1.3
Daubachies 4*	1.5
Daubachies 8*	1.6
Symmlet*	1.6
No Transform	1.9



Conclusions

- Applying transforms to the inputs can result in faster convergence for the adaptation weights.
  - Inputs are decorrelated
  - Transformed inputs have equal power
  - Faster convergence occurs when inputs have equal power
- There exist many possible transforms can improve the speed of the equalizer.
- However, it is difficult to demonstrate the superiority of one transform over another.
- Applying transforms on the inputs can be computationally complex.
  - More power
  - More hardware
  - More cost

References

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- *Beaufays F.* ***Transform-domain adaptive filters****: an analytical approach. [Journal Paper] IEEE Transactions on Signal Processing, vol.43, no.2, Feb. 1995, pp.422-31. USA.*